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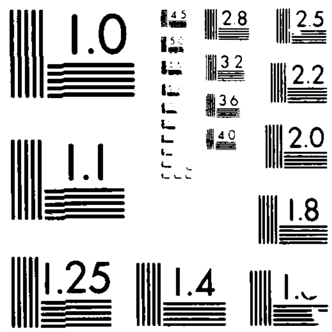

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Some fairly different types of self-adaptive grid refinement techniques were considered: 1)local refinement on a fixed mesh, 2)adaptive movement of the computational mesh to give better resolution of changing local phenomena, 3)dynamic, element-by-element refinement of an initial coarse grid, and 4)patch uniform refinement which also can be adapted to changing phenomena. One objective of this research was to develop fairly general adaptive techniques which are efficient for time-dependent problems and capable of being incorporated easily in large-scale simulation. Numerous papers and talks were produced under this effort; representative titles included "A preconditioning technique for the efficient solution of problems with local grid refinement", "Adaptive grid refinement methods for time-dependent flow problems", "Applications of the moving finite element method for systems in 2-D", "Moving finite element solution of systems of partial differential equations with methods to control model positious", and "Applications of the moving finite element method to moving boundary Stefan problems".

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AFOSR-TR- 87-1984

FINAL TECHNICAL REPORT

Adaptive Local Grid Refinement in Computational Fluid Mechanics

Air Force Grant No. AFOSR-85-0117

submitted by:

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November, 1987

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TABLE OF CONTENTS

SUMMARY

Several promising new techniques for efficient and accurate numerical solution of large-scale fluid flow problems have been developed. These methods include self-adaptive mesh modification techniques for applications requiring front-tracking and local grid refinement as well as new preconditioning ideas for efficient implementation. Properties of systems of hyperbolic conservation laws have been obtained which will aid in development of accurate front-tracking algorithms. Adaptive grid refinement techniques developed include moving grid methods, local, fixed refinement involving linked-list data structures, and certain local patch refinement ideas which have great potential for ease of implementation in existing large scale codes. Domain decomposition concepts for obtaining efficient preconditioners for iterative techniques have proved quite useful both for local patch refinement and for the solution of problems with rapidly varying coefficients. The fast adaptive algorithms being developed have high potential for both parallelization and vectorization. The algorithms have been designed to take advantage of the emerging parallel and vector-oriented computer architectures. Finally, the incorporation of these fast algorithms in accurate finite element, collocation, and finite difference methods is underway.

RESEARCH OBJECTIVES

Many phenomena in large-scale fluid dynamics problems are sufficiently localized that self-adaptive local grid refinement techniques are essential for their numerical simulation. These simulations involve the numerical solution of large coupled systems of nonlinear partial differential equations. In the long run, efficient, self-adaptive refinement methods must be developed for three-dimensional, fluid flow problems with important transient character. The research objectives for this project were to develop efficient and accurate numerical modeling techniques for large fluid flow applications involving efficient preconditioning concepts and self-adaptive local grid refinement methods.

We considered four fairly different types of self-adaptive grid refinement techniques: 1) local refinement on a fixed mesh, 2) adaptive movement of the computational mesh to give better resolution of changing local phenomena, 3) dynamic, element-by-element refinement of an initial coarse grid, and 4) patch uniform refinement which also can be adapted to changing phenomena. One objective of this research was to develop fairly general adaptive techniques which are efficient for time-dependent problems and capable of being incorporated easily in large-scale simulation. Grid modification methods associated with finite element, collocation and finite difference discretization were studied.

Adaptive gridding methods are also required to locate and follow shocks or near-shocks. Research has been aimed at shock tracking or shock capturing techniques. Although many applications require the tracking of discontinuities or interfaces, the common feature of these problems is geometry. One research goal was to develop software to handle such geometries in a manner which is basically independent of the physics of the applications, if possible. Since hyperbolic conservation laws are the equations which give rise to shocks, this research also involves study of Riemann problems for hyperbolic equations.

STATUS OF THE RESEARCH

A. Adaptive and Local Grid Refinement Techniques

The different grid refinement techniques that we have been studying are described and compared briefly in references [1–4, 6–8, 10, 17–20, 27–28, 31–34, 36–39, 41, 43, 45]. The survey presented in [18] compares some of these techniques with different methods that are being considered by others. References [3,6,16–21, 25, 31] illustrate how some of these techniques are being applied to certain fluid dynamics problems. We discuss four different approaches to grid refinement below.

The first method that we have studied yields truly local grid refinement capabilities, where an arbitrary level of refinement can be applied in an arbitrary region or at an arbitrary point in space or time. A multilinked tree data structure has been developed for efficient matrix set-up and solution. The dynamic multilinked list representation efficient, allows both placement of mesh refinement and the removal of local meshes.

The data structure supports a grid refinement capability having a set of predetermined macro-cells. Each macro-cell can be locally subdivided by a repeated nested refinement into local elements or cells. The tree structure describes the local cell arrangements. A global forest-like structure describes the macro-cell interrelationships. Since the individual trees can change dynamically, factorization of the codes is very difficult. Parallelization at the forest and lower levels in the trees has been addressed in [10, 28]. Implementation of these algorithms on the Alliant FX/8 Parallel Architecture, purchased through a DOD grant for our new Institute for Scientific Computation, is underway.

The second type of adaptive techniques under study involves the movement of grid nodes in time to "optimal" locations for the numerical approximations. Moving finite element (MFE) methods have been developed and extended by Miller, Djomehri and colleagues [4, 40, 41, 43, 45]. Djomehri has modified his MFE code in one space dimension (1–D) and has applied it to several partial differential equations (PDEs) of hyperbolic and/or parabolic type. The code was implemented to study the behavior of the solution of a model problem in the nonlinear instability analysis of a modified form of Burgers' equation in a joint paper with Straughan, Ewing, Djomehri and Jacobs [31]. This was a prototype of a more complicated problem about three dimensional flows for an incompressible fluid with highly nonlinear effects. The competition and interaction between various dominating terms such as convection, diffusion, and nonlinear source terms in the equation are responsible for generating sharp boundaries and interior layers. The MFE method was employed using only 20–30 moving grid points for this problem, yielding very accurate results. The Burgers' equation examples were also solved by the fixed finite element (FFE) algorithm for the comparison. The FFE approach requires at least 1000 fixed grid points to achieve the same order of accuracy obtained by the MFE method [31]. Moreover, it has been observed that for rather thin boundary layers the FFE approach becomes unstable.

Djomehri and George have also applied the MFE method to moving boundary problems. Several versions of the classical ice-water Stefan problem have been solved in one space dimension. The Stefan problem provides an excellent test case for numerical methods. In these problems the spatial domain occupied by the fluid deforms as a function of time. The state of the fluid near the boundary determines the moving boundary. The MFE code has been modified and successfully accounts for the timewise deforming spatial domain solutions in one-dimension. A recent paper submitted by George and Djomehri [43] further discusses and compares the solution of several interesting problems with preexisting results.

Djomehri has also applied the MFE code to nonlinear shallow water problems in one dimension to simulate the "dam-break" problem with step-function initial conditions. After the break, both a shock wave and a rarefaction wave propagate in different directions, reflect from the boundaries, and intract again. The MFE method has resolved the nonlinear hyperbolic shallow water waves quite accurately in comparison with the exact and other numerical solutions. Results will appear in a forthcoming paper by Djomehri [45].

The third approach to local grid refinement involves dynamic, element-by-element refinement of an initial coarse grid. Allen is currently working on this type of refinement scheme for advection-dominated flows. He has worked with a graduate student, Mark Curran, on the development of a general code to solve quasilinear equations of order three or less. This general form includes the Buckley-Leverett equation (see [12, 13]), Burgers' equation, the inviscid Burgers' equations, nonlinear reaction-diffusion equations, and soliton-producing equations of the Korteweg-deVries type. The aim of this effort is to develop stable computational procedures that include adaptive local grid refinement in Hermite cubic trial spaces. To date this research has yielded an h-adaptive refinement scheme for linear advection-diffusion equations allowing arbitrary refinement in any coarse-grid element. A report of this scheme will appear in a paper by Allen and Curran [38]. Extensions of this method to the nonlinear Burgers' equation are near completion and will appear in another paper by Allen and Curran [39]. This work will provide a basis for comparison and further analysis with the previously described MFE codes for Burgers' equation developed by Djomehri *et al.*

The fourth type of adaptive refinement, is patch refinement. Here, patches are chosen in regions of interest and uniform refinement is performed on the patches. This local refinement is much easier to vectorize than truly local refinement. In addition, each patch can be sent to a different processor to achieve parallelism in the algorithms. Ewing has interacted with McCormick and Thomas from neighboring Colorado schools in a technique called the fast adaptive composite (FAC) grid technique. This method, involving a local multigrid solver on the patches, is described in [2]. Ewing is also working with Bramble, Pasciak and Schatz [32] on some exciting new techniques for identifying suitable preconditioners to tie a local patch solution to a global solution process efficiently without disrupting the basic algorithms and codes. There is extensive potential in these methods to combine local refinement methods with existing large-scale codes. These ideas are similar to domain decomposition techniques, which many investigators have found useful in segmenting large problems. These methods have great potential for parallel algorithm development and will be implemented on our new parallel architecture Alliant FX/8.

The patch adaptive refinement techniques developed in [32] have been combined with operator splitting methods and applied to fluid flow problems by Ewing and coworkers in [28, 29, 33, 37]. The operator-splitting treats the hyperbolic part of a transport-dominated diffusion process via modified method of characteristic methods (see [29] and references in [29, 33, 37]) and the diffusive part via Galerkin or Petrov-Galerkin techniques.

B. Discretization and Efficient Iterative Solution Techniques

This general area of research comprises three related subjects: the exploitation of mixed finite element concepts for the development of highly accurate discretizations, the implementation of efficient iterative solvers for large linear systems, and the construction of stable iterative procedures for solving discrete systems arising from nonlinear problems.

Several papers have discussed our new developments in mixed finite element methods for obtaining accurate fluid velocities [9, 16, 19, 20, 24, 25]. These methods are very useful for applications with changing local flow properties or for various singularity-removal techniques [9, 16, 25]. We are currently trying to incorporate these methods in existing large-scale finite difference simulators for fluid flow problems [24].

The mixed finite element methods that we have developed utilize preconditioned conjugate-gradient iterative procedures to solve the linear algebraic systems generated by the discretization. We are studying a wide variety of ways to obtain better preconditioners that are applicable to a wide variety of problems. Obeysekare, Allen, George, Ewing, Koebbe, and Oliver have developed nested-factorization, generalized conjugate-residual codes and have applied them successfully to solve two-dimensional nonlinear hyperbolic conservation laws in [30]. This problem, which stems from a non self-adjoint operator, produces both strongly asymmetric matrix equations and shocks. These are both difficult problems that can be treated efficiently via these methods.

In a more recent investigation of iterative linear solution techniques, Allen and Ewing are working with a graduate student (Peng Lu) to investigate smoothers of the block-iterative type that will be appropriate for multigrid solutions of mixed finite-element discretizations. Earlier work on this grant (see, for example [9]) has demonstrated the effectiveness of mixed methods for such problems as determining underground fluid velocities. The convergence properties of the block-iterative schemes are now understood in some cases, and ongoing work focuses on coding and testing their residual smoothing properties.

A different line of research on iterative techniques concerns the iterative solution of the nonlinear algebraic equations that result from the discretization of nonlinear PDEs. In this vein, Allen has developed finite-element collocation solutions to nonlinear wave problems. This work has led to a successful class of Newton-like techniques for implicitly solving problems of the nonlinear heat-equation variety that arise in simulating transient flows in unsaturated porous media. These techniques yield stable, mass-conserving formulations of the finite-element collocation method in problems involving sharp wetting fronts. Results of this work appear in Allen and Murphy [11, 26] and Murphy and Allen [21]. Related work by M.A. Celia at M.I.T. has borrowed this iterative approach to advantage in alternating-direction collocation schemes, which promises even greater computational efficiency. Allen and Celia have recently begun considering the adaptation of these techniques to parallel computing environments.

C. Hyperbolic Equations and Conservation Laws

Eli Isaacson has been working on both computational and analytical aspects of Riemann problems for conservation laws. These problems arise in combustion, mach stem and various shock calculations, transonic flow around airfoils, multiphase flows in porous media, and tracking discontinuities in simulations. The main computational work is the development, with D. Marchesin and B. Plohr, of a general purpose computer code to solve Riemann problems for 2×2 systems of conservation laws. This code is currently being incorporated in the general front tracking code of J. Glimm, O. McBryan, *et al.* The Riemann solver is needed to advance the front at each time step. A by-product of this work has been the development of a computer code that determines level surfaces of vector functions of several variables [46]. This code is being modified for vectorization and parallelization.

Analytical work completed with D. Marchesin, B. Plohr, and B. Temple concerns the classification of solutions of Riemann problems for hyperbolic conservation laws near an isolated singularity [5, 14, 15, 22]. The behavior of the shock and rarefaction waves near such a point is nonclassical. In fact, many new features in the structure of solutions of conservation laws have been discovered in these problems [49, 50]. This has led to the analysis of the structure of general wave curves for conservation laws [47]. In addition, a new type of shock wave seems to be necessary to be able to solve certain problems. However, it is now necessary to determine by a physical criterion when such shock waves are admissible. A case has been studied in which classical shock waves lead to a solution, but a different solution is obtained when the physical admissibility criterion of viscous profiles is applied [48].

Finally, work recently completed with B. Temple concerns the structure of asymptotic states for a specific conservation law [49]. This work indicates that the standard ideas about stability of shock waves should be modified to include additional physical information.

Allen has also done some work on a coupled system of hyperbolic PDE's governing gravity waves in shallow water bodies and large-scale atmospheric motions. This work uses a Galerkin finite-element procedure together with a quadrature technique capable of damping spurious waves of length $2\Delta x$ that afflict standard finite-difference methods. The objective of this research is to develop multigrid algorithms suitable for hyperbolic wave equations. So far, the multigrid algorithms that have been coded have led only to marginal improvements in CPU time over standard solution procedures. However, if the difficulties could be overcome, the resulting technique would have broad applications ranging from numerical weather prediction to seismic signal modeling.

The work with Sochacki *et al.* has developed some new concepts for dealing with absorbing boundary conditions [42]. We have developed a scheme for treating a curved surface that occurs in the grid as a result of discontinuous changes in material properties. This paper is being revised for *Geophysics*, and has applications to local refinement near the curved surface.

CHRONOLOGICAL BIBLIOGRAPHY

1. Adaptive mesh refinements in reservoir simulation applications (R.E. Ewing), *Proceedings Intl. Conference on Accuracy Est. and Adaptive Refinements in Finite Element Computations*, Lisbon, Portugal, June 19–22, 1984, pp.31–40.
2. The fast adaptive composite grid method for solving differential boundary-value problems (R.E. Ewing, S. McCormick and J. Thomas), *Proc. Fifth ASCE Specialty Conference "Engineering Mechanics in Civil Engineering,"* Laramie, Wyoming, August 1–3, 1984, pp. 1453–1456.
3. Local grid refinement for oil recovery simulation (J.C. Diaz, R.E. Ewing, R.W. Jones, A.E. McDonald, D.U. von Rosenberg and L.M. Uhler), *Proceedings of SEG/SIAM/SPE Conference on Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling*, Houston, Texas, January 21–24, 1985, (W.E. Fitzgibbon, ed.), SIAM Publications (1986) pp. 238–240.
4. The moving finite element method and its applications (M.J. Djomehri), *Proc. of SEG/SIAM/SPE Conference on Mathematical Modeling and Computational Methods in Seismic Exploration and Reservoir Modeling*, Houston, Texas, January 21–24, 1985, SIAM Publications (W.F. Fitzgibbon, Editor), pp. 57–61.
5. Examples and classification of solutions of non-strictly hyperbolic conservation laws (E.L. Isaacson and B. Temple), *Abstracts of the AMS* (January 1985), presented at the Annual Meeting of the AMS, Anaheim, California, January 1985.

6. Self adaptive local grid refinement for time-dependent two-dimensional simulation (J.C. Diaz, R.E. Ewing, R.W. Jones, A.E. McDonald, L.M. Uhler and D.U. von Rosenberg), *Finite Elements in Fluids, Vol. VI*, John Wiley and Sons, Ltd., 1985, 279–290.

7. Mixed finite element methods for accurate fluid velocities (R.E. Ewing, J.V. Koebbe, R. Gonzalez and M.F. Wheeler), *Finite Elements in Fluids, Vol. VI*, John Wiley and Sons, Ltd., 1985, pp. 233–249.

8. Adaptive local grid refinement (R.E. Ewing), *Proceedings SEG/SIAM/SPE Conference on Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling*, Houston, Texas, January 21–24, 1985, (W.E. Fitzgibbon, ed.), SIAM Publications (1986), pp. 235–247.

9. Mixed finite-element methods for computing groundwater velocities (M.B. Allen, R.E. Ewing and J.V. Koebbe), *NUMETA 85, Numerical Methods in Engineering: Theory and Applications, Volume II*, (J. Middleton and G.N. Pande, eds.), A.A. Balkema Publishers, Rotterdam, Netherlands, 1985, pp. 609–614; and *Numerical Methods for Partial Differential Equations, 3* (1985), pp. 195–207.

10. Potential of HEP-like MIMD architecture in self adaptive local grid refinement for accurate simulation of physical processes (R.E. Ewing and J.C. Diaz), *Proceedings Workshop on Parallel Processing Using the HEP*, Norman, Oklahoma, March 20–21, 1985, pp. 209–226.

11. A finite-element collocation method for variably saturated flows in porous media (M.B. Allen and C.L. Murphy), *Numerical Methods for Partial Differential Equations, 3* (1985), pp. 229–239.

12. Numerical modeling of multiphase flow in porous media (M.B. Allen), presented at the NATO Advanced Study Institute on Fundamentals of Transport Phenomena in Porous Media, Newark, Delaware, July 1985; also in *Advances in Water Resources, 8:4* (1985), pp. 162–187.

13. Author's reply to discussion of the convergence of upstream collocation in the Buckley-Leverett problem (M.B. Allen), *Society of Petroleum Engineers Journal*, December, 1985, pp. 943–944.

14. Classification of the solutions of quadratic Riemann problems, I (E.L. Isaacson, D. Marchesin. B. Plohr, and B. Temple), *PUC/RJ Report MAT 12-85* and *MRC Technical Summary Report #2891*, December 1985.

15. Classification of the solutions of quadratic Riemann problems, II (E.L. Isaacson and B. Temple), *MRC Technical Summary Report #3892*, December 1985.

16. Finite element methods for nonlinear flows in porous media (R.E. Ewing), *Computer Meth. Appl. Mech. Eng., 51* (1985), pp. 421–439.

17. Adaptive mesh refinement in large-scale fluid flow simulation (R.E. Ewing), *Accuracy Estimates and Adaptivity for Finite Elements*, Chapter 16,

Babuska, (O.C. Zienkiewicz and E. Oliveira, eds.), John Wiley and Sons, New York, 1986, pp. 299–314.

18. Efficient adaptive procedures for fluid flow applications (R.E. Ewing), *Computer Meth. Appl. Mech. Eng., 55* (1986), pp. 89–103.

19. Finite element techniques for reservoir simulation (R.E. Ewing and J.V. Koebbe), *Innovative Numerical Methods in Engineering*, (R.P. Shaw, J. Periaux, A. Chaudouet, J. Wu CiMerino and C.A. Brebbia, eds.), Springer-Verlag, Berlin, 1986. pp. 173–186.

20. Variational methods for fluid flow in porous media (R.E. Ewing), *Variational Methods in Geosciences*, (Y. Sasaki, ed.), Elsevier, Amsterdam, 1986, pp. 251–263.

21. A collocation model of two-dimensional unsaturated flow (C.L. Murphy and M.B. Allen), in *Proceedings, Sixth International Conference on Finite Elements in Water Resources*, Lisbon, Portugal, June 1–5, 1986, (A. Sá da Costa *et al.*, eds.), Berlin: Springer-Verlag, 411–420.

22. Classification of the solutions of quadratic Riemann problems, III (E.L. Isaacson and B. Temple), *MRC Report*, 1986.

23. Geometrical aspects of sorted patterned ground in recurrently frozen soil (K.J. Glason, W.B. Krantz, N. Cains, J.H. George and R.D. Gunn), *Science, 232* (1986), pp. 216–220.

24. Accurate velocity weighting techniques (R.E. Ewing, J.V. Koebbe and R. Lagnado), *Proceedings Second Wyoming Enhanced Oil Recovery Symposium*, Casper, Wyoming, May 15–16, 1986, pp. 140–157.

25. Mathematical modeling and large-scale computing in energy and environmental research (R.E. Ewing), in *New Directions in Applied and Computational Mathematics* (R.E. Ewing, K.I. Gross and C.F. Martin, eds.), Springer-Verlag, Berlin, 1986 pp. 45–60.

26. A finite-element collocation method for variably saturated flow in two space dimensions (M.B. Allen and C.L. Murphy), *Water Resources Research, 22:11* (1986), 1537-1542.

27. Numerical solution of systems of partial differential equations (R.E. Ewing), *Transactions of Fourth Army Conference on Applied Mathematics and Computing*, ARO Report 87–1, 1987, pp. 583–595.

28. Simulation techniques for multiphase and multicomponent flows (R.E. Ewing, M.S. Espedal, J.A. Puckett and R.S. Schmidt), *Proceedings of Workshop on Special Topics in Computational Mechanics*, Dallas, Texas, April 13-14, 1987.

29. Finite element methods for contamination by nuclear waste-disposal in porous media (R.E. Ewing, Y. Yuan and G. Li), *Proceedings of Dundee Numerical Analysis Conference*, Dundee, Scotland, June 23–26, 1987.

30. Application of conjugate gradient-like methods to a hyperbolic problem in porous media flow (U. Obeysekare, M.B. Allen, R.E. Ewing and J.H. George), *International Journal for Numerical Methods in Fluids, 7* (1987), pp. 551–566.

31. Nonlinear instability for a modified form of Burgers' equation (R.E. Ewing, B. Straughan, P.G. Jacobs and M.J. Djomehri), *Numerical Methods for Partial Differential Equations, 3* (1987) pp. 51–64.

32. A preconditioning technique for the efficient solution of problems with local grid refinement (J. Bramble, R.E. Ewing, J. Pasciak and A. Schatz), *Computer Methods in Applied Mechanics and Engineering*, (to appear).

33. Characteristic Petrov-Galerkin subdomain methods f r two-phase immiscible flow (M. Espedal and R.E. Ewing), *Computer Methods in Applied Mechanics and Engineering*, (to appear).

34. Adaptive grid refinement methods for time-dependent flow problems (R.E. Ewing), *Communications in Applied Numerical Methods*, (to appear).

35. Velocity weighting techniques for fluid displacement problems (R.E. Ewing, R.F. Heinemann, J.V. Koebbe and U.S. Prasad), *Computer Methods in Applied Mechanics and Engineering, 64* (1987), pp. 137–151.

36. Adaptive grid-refinement techniques for treating singularities, heterogeneities and dispersion (R.E. Ewing), *Proceedings of Symposium on Numerical Simulation in Oil Recovery*, Minneapolis, Minnesota, Springer-Verlag, Berlin, (to appear).

37. Characteristic Petrov-Galerkin subdomain methods for convection diffusion problems (H. Dahle, M. Espedal and R.E. Ewing), *Proceedings of Symposium on Numerical Simulation in Oil Recovery*, Minneapolis, Minnesota, Springer-Verlag, Berlin, (to appear).

38. An adaptive gridding scheme for solving advection-dominated flows using finite-element collocation (M.B. Allen and M.C. Curran), to appear in *Proceedings, International Conference on Computational Engineering Science*, Atlanta, George, April 10–14, 1988, (S. Atluri, ed.).

39. An adaptive collocation scheme for Burgers' equation (M.B. Allen and M.C. Curran), in preparation for *Proceedings, Seventh International Conference on Computational Methods in Water Resources*, Boston, Massachusetts, June 13–17, 1988, (M.A. Celia, *et al.*, eds.).

40. Applications of the moving finite element method for systems in 2-D (M.J. Djomehri, S. Doss, R.J. Galinas and K. Miller), *J. Comp. Phys.*, (submitted).

41. Moving finite element solution of systems of partial differential equations with methods to control nodal positions (M.J. Djomehri), *SIAM J. on Scientific and Statistical Analysis*, (submitted).

42. Absorbing boundary conditions and surface waves (J. Sochacki, R. Kubichek, J.H. George, S. Smithson and W. Fletcher), *Geophysics, 52* (1987), pp. 60–71.

43. Application of the moving finite element method to moving boundary problems (J. George and J. Djomehri), *Computer Methods in Applied Mechanics and Engineering*, submitted.

44. A comparison of maximum entropy and maximum likelihood methods for directional wave analysis (J.H. George and L. Borgman), (in preparation).

45. Application of the moving finite element method to moving bounary Stefan problems (J. Djomehri), (in preparation).

46. An algorithm for finding level surfaces of vector functions of several variables (E. Issacson and D. Marchesin), in preparation.

47. The structure of wave curves for conservation laws (E. Issacson, D. Marchesin and B. Plohr), in preparation.

48. Viscosity profiles for transitional shock waves (E. Isaacson, D. Marchesin and B. Plohr), in preparation.

49. The structure of singular conservation laws (E. Isaacson, D. Marchesin, B. Plohr and B. Temple), submitted.

50. Asymptotic states for a singular conservation law (E. Isaacson and B. Temple), preprint.

51. The classification of solutions of quadratic Riemann problems I (E. Isaacson, D. Marchesin, B. Plohr and B. Temple), submitted.

52. The classification of solutions of quadratic Riemann problems II (E. Isaacson and B. Temple), to appear.

53. The classification of solutions of quadratic Riemann problems III (E. Isaacson and B. Temple), to appear.

INVITED TALKS

R.E. Ewing

1. Parameter estimation in large-scale simulation, Institute for Computation in Science and Engineering, NASA Langley, Hampton, Virginia, June 10–14, 1985.

2. Efficient adaptive procedures for fluid flow applications, Symposium on Computational Mechanics, Second Joint ASCE/ASME Mechanics Conference, Albuquerque, New Mexico, June 23–26, 1985.

3. Status of the oil and gas industry, Visiting Professor, Pacific Power and Light Company, Portland, Oregon, August 7, 1985.
4. Mathematical modeling and large-scale computing in energy and environmental research, Conference on New Directions in Applied and Computatinal Mathematics, Laramie, Wyoming, August 8–10, 1985.
5. Mathematics and engineering in large-scale computational science, National Science Foundation, Washington, D.C., October 11, 1985.
6. Project type and multi-agency grants, 1985 National Chairmen's Research Colloquium for the Mathematical Sciences, Washington, D.C., October 1985.
7. Variational methods for petroleum reservoir simulation, Keynote Address, Internatinoal Symposium on Variational Methods in Geosciences, Cooperative Institute for Mesoscale Meterological Studies, Norman, Oklahoma, October, 14–17, 1985.
8. Large-scale simulation in enhanced oil recovery research, Annual meeting Wyoming Colleges and Universities, Cheyenne, Wyoming, February 21, 1986,
9. Finite element techniques for reservoir simulation, Fourth International Symposium on Numerical Methods for Engineers, Atlanta, George, March 24–28, 1986.
10. Adaptive local grid refinement, Third Stanford Reservoir Simulation Workshop Program, Stanford, California, March 31–April 1, 1986.
11. Introduction and overview of the Enhanced Oil Recovery Institute, Second Wyoming Enhanced Oil Recovery Symposium, Casper, Wyoming, May 15–16, 1986.
12. Parameter estimation for fluid flow problems, Inverse Problems, Oberwolfach, West Germany, May 18–24, 1986.
13. Numerical solution of partial differential equations, Fourth Army Conference on Applied Mathematics and Computing, Ithaca, New York, May 27–30, 1986.
14. Mathematical modeling in the energy and environmental sciences, (Series of 10 invited lectures) Principal Lecturer at NSF-CBMS Regional Conference, Morgantown, West Virginia, June 2–6, 1986.
15. Estimation of spatially dependent parameters in parabolic partial differential equations, Alpine–U.S. Seminar on Inverse and Ill-posed Problems, St. Wolfgang, Austria, June 8–13, 1986.
16. Identification of parameters in distributed systems, Conference on Control and Identification of Distributed Systems, Vorau, Austria, July 6–12, 1986.
17. Adaptive local grid refinement, 3rd Mexican-American Conference on Computational Modeling in Science and Engineering, Avandaro, Mexico, July 22, 1986.
18. Modeling of multiphase contaminant flows, Forum on NSF Research Activities in Subsurface Systems, Ann Arbor, Michigan, July 24, 1986.

19. Domain decomposition method for adaptive local grid refinement, Workshop on Preconditioned Iterative Methods and Domain Decomposition, Mathematical Sciences Institute, Ithaca, New York, August 12, 1986.

20. Adaptive grid refinement methods for time dependent flow problems, First World Congress on Computational Mechanics, University of Texas, Austin, Texas, September 25, 1986.

21. Velocity weighting techniques for fluid displacement problems, First World Congress on Computational Mechanics, University of Texas, Austin, Texas, September 25, 1986.

22. Optimization techniques in reservoir simulation, Special Session on Optimization and Inverse Problems in Reservoir Aquifer Modeling, Operations Research Society of America ORSA/TIMS National Meeting, Miami Beach, Florida, October 28, 1986.

23. Large-scale computing in fluid flow problems, Special Session on Mathematics for Large-Scale Computing, 830th Meeting of the American Mathematical Society, Denton, Texas, October 31, 1986.

24. Techniques for treating heterogeneities and dispersion in reservoir simulation, Sympoisum on Numerical Simulation in Oil Recovery, Institute for Mathematics and Its Applications, University of Minnesota, Minneapolis, Minnesota, December 1-12, 1986.

25. The use of mixed finite element methods for accurate fluid velocities, Workshop on Recent Developments in Leaky Aquifer Mechanics, Instituto de Geofisica, Universidad Nacional de Mexico, Mexico City, Mexico, January 20-21, 1987.

26. Simulation techniques for multiphase and multicomponent flows, Invited Lecture Series, IBM Bergen Scientific Center, Bergen, Norway, March 23-24, 1987.

27. Parallel computation in operator splitting and self adaptive local grid refinement, Workshop on Special Topics in Computational Mechanics, Dallas, Texas, April 13-14, 1987.

28. Status of the Enhanced Oil Recovery Institute, Third Wyoming Enhanced Oil Recovery Symposium, Casper, Wyoming, May 13-14, 1987.

29. Reservoir simulation at the University of Wyoming, Third Wyoming Enhanced Oil Recovery Symposium, Casper, Wyoming, May 13-14, 1987.

30. Mathematics and mathematical modeling in interdisciplinary research and large scale computation, Address to National Science Foundation, Washington, D.C., May 27, 1987.

31. Analysis and computation for a model for a model for possible contamination by nuclear waste-disposal in porous media, 12th Biennial Conference on Numerical Analysis, University of Dundee, Scotland, June 23-24, 1987.

32. Parameter estimation problems for parabolic problems, Workshop on Applications and Algorithms for Optimal Control and Parameter Identification, Universität Trier, Trier, West Germany, June 25-26, 1987.

33. A survey of reservoir simulation, Minisymposium on Simulation of Petroleum Reservoirs, First International Conference on Industrial and Applied Mathematics, Paris, France, June 29–July 3, 1987.

34. Techniques for multiphase and multicomponent flows, Fourth Mexican American Exchange in Mathematics and its Applications, Two Bars Seven Ranch, Colorado, August 3–6, 1987.

35. Nonlinear convection diffusion equations arising in multiphase flows, Nonlinear Parameter-Dependent PDE's and Their Effect Solution, Arizona State University, Tempe, Arizona, November 6, 1987.

36. Additional Invited Seminars on Numerical Reservoir Simulation and Large Scale Simulation:

    a. University of Houston, April 16, 1985.
    b. Colorado State University, April, 18, 1985.
    c. University of Colorado, Boulder, May 2, 1985.
    d. Pacific Power and Light Corp., Portland, Oregon, September 10, 1985.
    e. Chevron Oil Field Research Co., La Habra, California, December 17, 1985.
    f. University of Minnesota, Minneapolis, December 19, 1985.
    g. University of Wyoming Foundation, Houston, Texas, February 11, 1986.
    h. Chevron Oil Field Research Co., La Habra, California, March 27, 1986.
    i. INRIA, Le Chesnay, France, June 24, 1986.
    j. University of Colorado, Denver, Colorado, September 10, 1986.
    k. National University of Mexico, Mexico City, Mexico, September 29, 1986.
    l. Tulsa University, Tulsa, Oklahoma, October 29, 1986.
    m. University of Minnesota, Minneapolis, Minnesota, December 4, 1986.
    n. University of Wyoming, Laramie, Wyoming, February 10, 1987.
    o. Utah State University, Logan, Utah, February 12, 1987.
    p. University of Wyoming, Laramie, Wyoming, February 17, 1987.
    q. Koninklijke/Shell Exploratie en Produktie Laboratorium, Rijswijk, The Netherlands, March 9, 1987.
    r. Norsk Hydro Petroleum Research Centre, Bergen, Norway, March 20, 1987.
    s. Universitet Bergen, Bergen, Norway, March 23, 1987.
    t. IBM, Bergen Scientific Centre, Bergen, Norway, March 24, 1987.
    u. Christian Michelson Institute, Bergen, Norway, March 27, 1987.
    v. Uppsala University, Uppsala, Sweden, March 31, 1987.
    w. Chalmers University of Technology and The University of Göteborg, Göteborg, Sweden, April 1, 1987.
    x. University of California at Berkeley, Berkeley, California, April 29, 1987.
    y. Arizona State University, Tempe, Arizona, April 30, 1987.

z. Montana State University, Bozeman, Montana, May 21, 1987.

aa. Montana State University, Bozeman, Montana, May 22, 1987.

bb. DOE, Morgantown Energy Technology Center, Morgantown, West Virginia, July 31, 1987.

cc. Research Institute of Petroleum Exploration and Development, Beijing, People's Republic of China (4 talks), September 15-20, 1987.

dd. Nankai University, Tianjin, People's Republic of China (2 talks), September 21-23, 1987.

ee. Shandong University, Jinan, Shandong, People's Republic of China (3 talks), September 24-27, 1987.

ff. Xian Petroleum Institute, Xian, Shaanxi, People's Republic of China, September 29, 1987.

gg. Chengdu Branch of Academia Sinica, Chengdu, Sichuan, People's Republic of China, October 2, 1987.

M.B. Allen

1. Application of conjugate-gradient-like techniques to the two-dimensional Buckley-Leverett problem.
   a. Princeton—UNAM Workshop on Numerical Methods for PDE's, Princeton, New Jersey, May 14, 1986.
   b. NATO Advanced Study Institute on Porous Media, Newark, Delaware, July 22, 1985.
2. Numerical modeling of multiphase flow in porous media.
   a. NATO Advanced Study Institute on Porous Media, Newark, Delaware, July 22, 1983.
   b. Battelle Pacific Northwest Laboratories, Richland, WA, September 20, 1985.
   c. Colorado School of Mines, Golden, Colorado, November 8, 1985.
3. Collocation methods for computation of oil reservoir flows, Institute of Geophysics, National University of Mexico, Mexico City, January 15, 1985.
4. Mixed finite element methods for groundwater velocities, Institute of Geophysics, National Univeristy of Mexico, Mexico City, January 16, 1985.
5. On the technical basis of upwinding algorithms, Institute of Geophysics, National University of Mexico, Mexico City, January 17, 1985.
6. Application of multigrid methods to the shallow-water wave equations, Mexican-American Exchange in Mathematics and Applications, Avandaro, Mexico, July 25, 1986.
7. Basic mechanics of oil reservoirs (12-hour lecture series), IBM Bergen Scientific Institute, Bergen, Norway, October 6-10, 1986.

8. Quadrature-limited superconvergence in velocities computed by a mixed finite element method, Mexican-American Exchange in Mathematics and Applications, Laramie, Wyoming, August 3, 1987.

J.H. George

1. Preconditioned conjugate gradient methods, NAS Computational Group, Ames Research Center, NASA, Moffet Field, California, June 1987.
2. TVD schemes for solving hyperbolic PDE, Computational Fluids Group, Boulder, Colorado, October 1987.

E.L. Isaacson

1. Numerical analysis of the Glimm Scheme, Invited Address, Biennian Meeting of the Brazilian Mathematical Society, Pocos de Caldas, Brazil, July, 1985.
2. Non-strictly hyperbolic conservation laws, Mathematics Colloquium, University of South Carolina, April, 1986.
3. I.M.P.A. (Institsute for Pure and Applied Mathematics), Rio de Janeiro, Brazil, June, 1987.
4. Colloquia for the Department of Mathematics, University of St. Etienne, France, October 1987.
5. Colloquium for the Department of Mathematics, University of Nice, France, October, 1987.

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